

Trigonometry

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Labelling the Sides (relative to θ)

- **Hypotenuse (H):** Longest side; always opposite the right angle. Never changes.
 - **Opposite (O):** Side directly facing angle θ .
 - **Adjacent (A):** Side next to θ that is *not* the hypotenuse.
- Warning:** O and A **swap** when you change the reference angle. H never changes.

Finding a Missing Side

1. Label sides O, A, H relative to the given angle.
 2. Identify the two sides involved (known + unknown).
 3. Choose SOH, CAH, or TOA.
 4. **Unknown on top:** multiply by trig ratio. **On bottom:** divide.
- $\cos 36^\circ = \frac{x}{8} \Rightarrow x = 8 \cos 36^\circ = 6.47 \text{ cm}$ $\cos 42^\circ = \frac{7.3}{h} \Rightarrow h = \frac{7.3}{\cos 42^\circ} = 9.82 \text{ cm}$

The Three Ratios: SOH CAH TOA

$$\sin \theta = \frac{O}{H} \quad \text{SOH: Opposite} \div \text{Hypotenuse}$$

$$\cos \theta = \frac{A}{H} \quad \text{CAH: Adjacent} \div \text{Hypotenuse}$$

$$\tan \theta = \frac{O}{A} \quad \text{TOA: Opposite} \div \text{Adjacent}$$

Finding a Missing Angle

- Use **inverse trig** when two sides are known: $\theta = \sin^{-1}\left(\frac{O}{H}\right)$, $\theta = \cos^{-1}\left(\frac{A}{H}\right)$, $\theta = \tan^{-1}\left(\frac{O}{A}\right)$
1. Label sides relative to the *unknown* angle.
 2. Identify the two known sides, choose the correct ratio.
 3. Apply the inverse. Round to **1 d.p.** (IGCSE standard).
- Tip:** Third angle = $90^\circ - \theta$ (angles in triangle sum to 180°).

Exact Trigonometric Values — Must Memorise!

Angle	$\sin \theta$	$\cos \theta$	$\tan \theta$	Derived from
0°	0	1	0	Limiting case
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	Equilateral triangle halved: sides 1, $\sqrt{3}$, 2
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	Isosceles right triangle: legs 1, 1; hyp $\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	Equilateral triangle halved: sides 1, $\sqrt{3}$, 2
90°	1	0	undefined	Limiting case

any angle)

Pythagorean Identity: $\sin^2 \theta + \cos^2 \theta = 1$ (always true for any angle)

Angles of Elevation & Depression

- **Elevation:** Angle *upward* from horizontal to an object above.
 - **Depression:** Angle *downward* from horizontal to an object below.
 - **Key:** Elevation A $\rightarrow$ B = Depression B $\rightarrow$ A (alternate angles).
 - **Strategy:** Draw horizontal; form right triangle; usually use TOA.
- Example:** 58° elevation, 65 m away $\Rightarrow h = 65 \tan 58^\circ = 104.0 \text{ m}$

Bearings and Trigonometry

- Measured **clockwise from North**; always **three digits** (e.g. 045°).
 - Range: 000° to 360° .
 - **cos** $\rightarrow$ North/South; **sin** $\rightarrow$ East/West.
 - For bearings $> 90^\circ$: extract the interior triangle angle first.
- Example:** 053° , 9.4 km: N = $9.4 \cos 53^\circ = 5.7 \text{ km}$; E = $9.4 \sin 53^\circ = 7.5 \text{ km}$

Trigonometry in 3D Problems

1. Identify the right-angled triangle you need.
 2. Use **Pythagoras** to find an intermediate length (e.g. base diagonal).
 3. Apply **SOH CAH TOA** to find the required side or angle.
- Cuboid:** base diag $d = \sqrt{l^2 + w^2}$, then $\tan \theta = \frac{h}{d}$
- Pyramid:** half base diag = $\frac{s\sqrt{2}}{2}$, then $\tan \theta = \frac{\text{height}}{\text{half diag}}$

Pythagoras vs. Trigonometry: Decision Guide

Use Pythagoras	Use SOH CAH TOA	Right
Know 2 sides, find 3rd	Angle given or asked for	
No angle given/needed	Know angle + 1 side	
$a^2 + b^2 = c^2$	$\sin \theta, \cos \theta, \tan \theta$	
angle? No $\rightarrow$ Sine/Cosine Rule. Yes $\rightarrow$ Angle involved? No $\rightarrow$ Pythagoras. Yes $\rightarrow$ Trig.		

Common Traps & Quick Exam Reference

Traps to Avoid:

1. **Wrong labels:** Always label O, A, H *first*, then pick ratio.
2. **Wrong mode:** Calculator must be in **Degree** mode, not Radians.
3. **Multiply vs. Divide:** Unknown on top $\rightarrow$ multiply; bottom $\rightarrow$ divide.
4. **Wrong angle:** Use the *given* angle, never the right angle.
5. **Early rounding:** Keep full values in working; round only at the final step.

Keyword Quick Reference:

- “Find side” + angle $\Rightarrow$ SOH CAH TOA (multiply or divide)
- “Find angle” + two sides $\Rightarrow \sin^{-1}, \cos^{-1}, \text{ or } \tan^{-1}$
- “Exact value” / “no calculator” $\Rightarrow$ use the exact values table
- “Elevation/Depression” $\Rightarrow$ draw horizontal; usually **tan**
- “Bearing” $\Rightarrow$ clockwise from North; **sin**=E/W, **cos**=N/S
- “3D shape” $\Rightarrow$ base diagonal (Pythagoras) first, then trig