

Sine & Cosine Rules

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Labelling Convention

Angles: capital letters A, B, C .

Sides: lowercase of the *opposite* angle.

- Side a opposite A ; side b opposite B ; side c opposite C .
- **Key:** longest side is always opposite the largest angle.

The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- **Find side:** side on top. **Find angle:** sine on top.
- **Use when:** AAS (two angles + any side) or SSA (two sides + non-included angle).

The Cosine Rule & Area Formula

$$1. \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$4. \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$2. \quad b^2 = a^2 + c^2 - 2ac \cos B$$

$$5. \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$3. \quad c^2 = a^2 + b^2 - 2ab \cos C$$

$$6. \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$7. \quad \text{Area} = \frac{1}{2} ab \sin C \quad (C \text{ is the **included** angle between sides } a \text{ and } b; \text{ SAS only})$$

Use 1–3 (SAS): find *side*. Use 4–6 (SSS): find *angle*. Use 7 (SAS): find *area*. **Negative cosine** $\Rightarrow$ obtuse.

Decision Guide & Ambiguous Case: The Strategy

Given	Find	Rule
Two angles + any side (AAS)	Side	Sine Rule
Two sides + non-incl. angle (SSA)	Angle/side	Sine Rule*
Two sides + incl. angle (SAS)	Side or area	Cosine Rule†
All three sides (SSS)	Angle	Cosine Rule

*Check ambiguous case! †Area: use formula 7 above.

Ambiguous Case

SSA: $\sin \theta = \sin(180^\circ - \theta)$ may give **two** triangles.

Test $B_2 = 180^\circ - B_1$; reject if $A + B_2 \geq 180^\circ$.

Obtuse given angle $\Rightarrow$ one solution only.

Common Traps & Pitfalls

1. **The Angle Trap:** $\frac{1}{2}ab \sin C$ requires C to be the angle **between** sides a and b . Verify before substituting.
2. **The Ambiguous Trap:** When finding an angle from SSA, always check whether $180^\circ - \theta$ also produces a valid triangle.
3. **The Sign Trap:** Obtuse angle $\Rightarrow$ negative cosine $\Rightarrow$ the $-2bc \cos A$ term becomes *positive*, making a^2 larger. This is geometrically correct.
4. **The Rounding Trap:** Keep full precision; only round at the final step — IGCSE mark schemes penalise premature rounding.

Quick Exam Shortcuts

Multi-step: Split shapes; carry exact values forward without rounding. **Largest angle first (SSS):** Find it first (no ambiguous case risk), then Sine Rule. **Sanity:** Angles = 180° , area > 0 , longest side opp. largest angle.

The Panda Math Tutor