

The Quadratic Formula — Prepared by Nikolas Agisilaou

Standard Form of a Quadratic Equation

Every quadratic must be written as $ax^2 + bx + c = 0$, $a \neq 0$.

- a = coeff. of x^2 (never zero), b = coeff. of x , c = constant.
- **Always rearrange to zero first**, then read off a , b , c .
- **Watch signs:** if equation has $-7x$, then $b = -7$, not $+7$.

$$\begin{aligned} 5x^2 - 3x + 7 &= 0 & a = 5, b = -3, c = 7 \\ 4x^2 = 7x - 2 &\Rightarrow 4x^2 - 7x + 2 = 0 & a = 4, b = -7, c = 2 \\ 8x - 3x^2 = 5 &\Rightarrow 3x^2 - 8x + 5 = 0 & a = 3, b = -8, c = 5 \end{aligned}$$

The Quadratic Formula

For $ax^2 + bx + c = 0$ ($a \neq 0$):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Gives **two** solutions: use $+$ then $-$ separately.
- **Rhyme:** “neg b , plus/minus root b^2 minus $4ac$, all over $2a$.”
- **Derivation:** divide by a , complete the square, isolate x .
- Given in *some* exams — memorise it regardless.

The Discriminant $\Delta = b^2 - 4ac$ — Compute This FIRST

Condition	No. of Real Roots	Nature	Parabola	Finding unknowns:
$\Delta > 0$	2 distinct real roots	Unequal values	Crosses x-axis at 2 points	
$\Delta = 0$	1 repeated real root	Equal ($x = -b/2a$)	Touches x-axis at vertex only	
$\Delta < 0$	No real roots	Not real	Lies entirely above or below x-axis	

set $\Delta = 0$ (equal roots), $\Delta > 0$ (two roots), or $\Delta < 0$ (no real roots), then solve for the unknown coefficient.

Exact Surd Answers

Leave in surd form when Δ is **not** a perfect square.

1. Compute $\Delta = b^2 - 4ac$.
 2. Simplify $\sqrt{\Delta}$ by pulling out perfect square factors:
 $\sqrt{44} = \sqrt{4 \times 11} = 2\sqrt{11}$; $\sqrt{28} = 2\sqrt{7}$.
 3. Cancel the common factor with $2a$:
 $\frac{8 \pm 2\sqrt{11}}{2} = 4 \pm \sqrt{11}$.
- Rule:** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ but $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$.

Graphical Interpretation

Roots of $ax^2 + bx + c = 0$ = x-intercepts of $y = ax^2 + bx + c$.

- **Vertex x-coord:** $x_v = -\frac{b}{2a}$; substitute back for y_v .
- Axis of symmetry is **halfway** between the two roots.
- $a > 0$: U-shape, minimum point; $a < 0$: $\cap$ -shape, maximum.
- **Check roots:** sum = $-\frac{b}{a}$; product = $\frac{c}{a}$.

Choosing the Right Method

Method	Use when...	Quick test / hint
Factorisation	Two integers multiply to ac and add to b	$x^2 - 7x + 12$: need $\times = 12$, $+= -7 \rightarrow -3$ and $-4 \checkmark$
Completing the Square	Need vertex form; or $a = 1$ with b even	Gives $(x + \frac{b}{2})^2 = k$; elegant for derivations
Quadratic Formula	Always works ; non-factorisable, decimal or surd answer needed	Compute Δ first — if $\Delta < 0$, stop immediately.

Common Traps & Pitfalls

1. **Sign trap:** $b = -5$ means $-b = +5$ and $b^2 = (-5)^2 = 25$, not -25 .
2. **$\pm$ trap:** Write **both** roots — omitting one loses marks.
3. **Fraction trap:** The *entire* numerator divides by $2a$, not just $\sqrt{\Delta}$.
4. **Rearrangement trap:** Set equation to zero **before** reading a, b, c .
5. **Context trap:** Reject impossible answers (e.g. negative length) and **explain why**.

Quick Exam Shortcuts

Step order: standard form $\rightarrow a, b, c \rightarrow$ compute $\Delta \rightarrow$ substitute $\rightarrow$ simplify $\rightarrow$ check.

Decimals: Round **only** at the final step; carry full precision through.

Equal roots ($\Delta = 0$): single root is $x = -b/2a$, which equals the vertex x-coord.

Surd shortlist: $\sqrt{4}, \sqrt{9}, \sqrt{16}, \sqrt{25}, \sqrt{36}, \sqrt{49}, \sqrt{64}, \sqrt{100}, \sqrt{121}, \sqrt{144}$ know