

Sequences

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Key Definitions

A **sequence** is an ordered list of numbers following a rule.

- **Term:** Each value in the list $(a_1, a_2, a_3, \dots)$.
- **n th term (a_n) :** Formula to find any term directly from its position n .
- **Common difference (d) :** Constant added each time (arithmetic).
- **Common ratio (r) :** Constant multiplied each time (geometric).

How to Identify the Type

Calculate differences and ratios to classify:

- **1st differences constant** → **Linear** (arithmetic).
- **2nd differences constant** → **Quadratic**.
- **Constant ratio** between consecutive terms → **Geometric**.
- **Each term = sum of previous two** → Fibonacci-type.

The Three Core n th Term Formulas

Linear	$a_n = a + (n - 1)d = dn + c$	$d = \text{common diff, } c = a - d$
Quadratic	$a_n = an^2 + bn + c$	$a = \frac{\text{2nd difference}}{2}$, then find $bn + c$
Geometric	$a_n = a \cdot r^{n-1}$	$a = \text{first term, } r = \text{common ratio}$

Finding the n th Term: Step-by-Step

Linear Sequence

1. Find $d = a_2 - a_1$
2. Write $a_n = dn + c$
3. Substitute $n = 1$ to find c
4. Check with another term

Tip: Coefficient of n = common difference.

Quadratic Sequence

1. Find 1st diffs, then 2nd diffs
2. n^2 coeff = $\frac{\text{2nd diff}}{2}$
3. Subtract an^2 from each term
4. Remainder is linear — find its n th term
5. Combine & check

Special Sequences to Recognise

Square numbers:	1, 4, 9, 16, 25, 36, 49, ...	$a_n = n^2$
Cube numbers:	1, 8, 27, 64, 125, 216, ...	$a_n = n^3$
Triangular numbers:	1, 3, 6, 10, 15, 21, 28, ...	$T_n = \frac{n(n+1)}{2}$
Powers of 2:	2, 4, 8, 16, 32, 64, 128, ...	$a_n = 2^n$
Fibonacci:	1, 1, 2, 3, 5, 8, 13, 21, ...	$a_n = a_{n-1} + a_{n-2}$

Common Traps & Pitfalls

1. **The Exponent Trap:** Geometric uses r^{n-1} , **not** r^n . The 5th term uses r^4 .
2. **The Halving Trap:** n^2 coefficient is **half** the 2nd difference. $2\text{nd diff} = 6 \Rightarrow \text{coeff} = 3$.
3. **The Membership Trap:** Solve $a_n = k$ for n . Only valid if n is a **positive integer**.
4. **The Sign Trap:** Negative d = **decreasing** sequence. Watch signs when computing c .
5. **The Subtraction Trap:** Subtracting an^2 for quadratics — double-check negatives.

Quick Exam Shortcuts

Is k in the sequence? Set $a_n = k$, solve for n . Positive integer $\Rightarrow$ yes; otherwise $\Rightarrow$ no.

Missing terms: Arithmetic: $d = \frac{a_{\text{last}} - a_{\text{first}}}{\text{gaps}}$. Geometric: $r = \sqrt[\text{gaps}]{\frac{a_{\text{last}}}{a_{\text{first}}}}$.

Two sequences crossing: Set n th terms equal, solve, and test nearby integer values.

Know these: Squares to $15^2=225$. Cubes to $6^3=216$. Triangulars: 1, 3, 6, 10, 15, 21, 28, 36.

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