

Ratio, Rate & Proportion

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Ratio

Definition: Comparison of *same-unit* quantities — **no units** in the answer.

Simplify:

- Convert all quantities to the **same unit** first
- Divide every part by the **HCF**
- Decimals: $\times 10^n$ to clear Fractions: $\times$ LCM of denominators
 $24 : 36 \xrightarrow{\div 12} 2 : 3$ $1.5 : 4.5 \xrightarrow{\times 10} 15 : 45 \xrightarrow{\div 15} 1 : 3$
 $\frac{3}{4} : \frac{5}{8} \xrightarrow{\times 8} 6 : 5$

Dividing Q in ratio $a : b$ (or $a : b : c$):

1. Total parts = $a + b$ (add all parts for three-way)
2. One part = $Q \div (a + b)$
3. Each share = ratio number $\times$ one part

Missing value: Divide known amount by *its* ratio number to get one part; multiply by the other.

Rate & Speed–Distance–Time

Rate: Comparison of *different-unit* quantities — answer **carries units**.

Unit rate: Divide to find value per 1 unit. Compare by finding cost per ml, per g, etc.

$D = S \times T$ find **distance**

$S = D \div T$ find **speed**

$T = D \div S$ find **time**



Cover the unknown!

Unit check: Speed in km/h
 $\Rightarrow$ time in **hours**, dist. in **km**.

Convert: mins $\div 60$ = hours.

Kinematic Graphs

	Distance–Time (d–t)	Velocity–Time (v–t)
Gradient equals	Speed (velocity)	Acceleration
Area under equals	—	Distance travelled
Horizontal line	Object stopped	Constant speed (not stopped) trapezoid shapes:
Straight slope	Constant speed	Constant acceleration Triangle (accel. from/to 0): $\frac{1}{2}bh$
Curve upward	Speeding up	Increasing acceleration Rectangle (constant velocity): bh
Downward slope	Returning to start	Decelerating Trapezium (between two speeds): $\frac{1}{2}(a+b)h$
		Acceleration = $\frac{\Delta v}{\Delta t}$
		Speed = $\frac{\Delta d}{\Delta t}$

Direct Proportion

$y \propto x \Rightarrow \boxed{y = kx}$ graph: **straight line through origin**

$\frac{y}{x} = k$ is **constant** (more $x \rightarrow$ more y)

Other forms:

$$y \propto x^2 \Rightarrow y = kx^2 \quad y \propto x^3 \Rightarrow y = kx^3$$

$$y \propto \sqrt{x} \Rightarrow y = k\sqrt{x}$$

Method: Write $y = kx^n \rightarrow$ sub. known pair to find $k \rightarrow$ use equation.

Inverse Proportion

$y \propto \frac{1}{x} \Rightarrow \boxed{y = \frac{k}{x}}$ graph: **hyperbola** (never touches axes)

$x \times y = k$ is **constant** (more $x \rightarrow$ less y)

Other forms:

$$y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2} \quad y \propto \frac{1}{\sqrt{x}} \Rightarrow y = \frac{k}{\sqrt{x}}$$

Method: Write $y = k/x^n \rightarrow$ find $k = x^n \times y \rightarrow$ use equation.

Percentage as Proportion

$$p\% = \frac{p}{100} \quad (\text{percentage is a ratio out of 100})$$

$$\% \text{ of a quantity: } \frac{p}{100} \times Q$$

$$\text{Express as \%: } \frac{\text{part}}{\text{whole}} \times 100$$

$$\% \text{ change: } \frac{\text{change}}{\text{original}} \times 100$$

$$\text{Increase by } r\%: \text{ multiply by } (1 + r/100)$$

$$\text{Decrease by } r\%: \text{ multiply by } (1 - r/100)$$

$$\text{Reverse \%: } \text{original} = \frac{\text{new value}}{\text{multiplier}}$$

Map Scales & Scale Drawings

$$\text{Scale} = \frac{\text{drawn length}}{\text{actual length}}$$

Scale 1 : n means 1 unit drawn $\equiv n$ units real.

$$\text{Actual} = \text{map} \times n$$

$$\text{Map} = \text{actual} \div n$$

Applying a scale factor k :

Length	Area	Volume
$\times k$	$\times k^2$	$\times k^3$

e.g. scale 1 : 40: area factor = $40^2 = 1600$, volume factor = $40^3 = 64\,000$

Common Traps & Pitfalls

- Units Trap:** Convert to same unit *before* writing a ratio.
50 cm : 2 m = 50 : 200 = 1 : 4, **not** 50 : 2.
- Direct vs Inverse:** Direct $\Rightarrow$ ratio $y/x = k$. Inverse $\Rightarrow$ product $xy = k$. Mixing these up gives a completely wrong answer.
- Graph Trap:** Horizontal on $d-t$ = **stopped**. Horizontal on $v-t$ = **constant speed**. Same shape; opposite meanings!
- Area Trap:** Area under $v-t$ gives **distance (metres)**. Decompose into triangles, rectangles, trapezia — do **not** count squares.
- Reverse % Trap:** To recover the original, **divide** by the multiplier — never subtract the % from the new value.
- Scale Factor Trap:** Length factor k gives area k^2 and volume k^3 . Never apply k to all three.

Quick Exam Keywords

- "Share / split / in ratio $a : b$ " $\rightarrow$
Ratio division: find one part first.
- "Per / each / best value" $\rightarrow$
Unit rate: divide to per-1-unit.
- "More... more" / "directly prop." $\rightarrow$
Direct: $y = kx^n$, find k from pair.
- "More... less" / "inversely prop." $\rightarrow$
Inverse: $xy = k$, use $y = k/x^n$.
- "Find original after % change" $\rightarrow$
Reverse %: new $\div (1 \pm r/100)$.
- "Gradient" $\rightarrow$ d-t: speed; v-t: accel.
- "Area under" $\rightarrow$ v-t only: distance.
- "Map / scale / model" $\rightarrow$
Scale 1 : n ; area k^2 ; volume k^3 .

Key Formulas at a Glance

Ratio	Proportion	Kinematics	Percentage
One part = $Q \div (a+b)$	Direct: $y = kx^n, \frac{y}{x} = k$	$S = \frac{D}{T}, D = ST, T = \frac{D}{S}$	% change = $\frac{\Delta}{\text{orig}} \times 100$
Simplify by HCF	Inverse: $y = \frac{k}{x^n}, xy = k$	Accel = $\frac{\Delta v}{\Delta t}$	Reverse: orig = $\frac{\text{new}}{1 \pm r}$
Scale: drawn $\div$ actual	Find k from a known pair	Area under v-t = distance	$p\% = p/100$