

Polygons

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What is a Polygon?

A **polygon** is a **closed**, flat shape made of **straight** line segments that do not cross each other.

- **Sides:** The straight line segments (edges).
- **Vertices:** The points where two sides meet.
- **Rule:** Number of sides = number of vertices = number of angles.

Naming Polygons

- | | |
|------------------------|--------------------------------|
| 3 Triangle | 8 Octagon |
| 4 Quadrilateral | 9 Nonagon |
| 5 Pentagon | 10 Decagon |
| 6 Hexagon | 12 Dodecagon |
| 7 Heptagon | n n -gon |

Regular = all sides equal AND all angles equal.

Irregular = sides or angles (or both) differ.

The Five Key Polygon Formulas

For any polygon with n sides:

- | | |
|---|---|
| 1. Sum of interior angles = $(n - 2) \times 180^\circ$ | 4. Sum of exterior angles = 360° |
| 2. Each interior angle (regular) = $\frac{(n - 2) \times 180^\circ}{n}$ | 5. Number of diagonals = $\frac{n(n - 3)}{2}$ |
| 3. Each exterior angle (regular) = $\frac{360^\circ}{n}$ | |

Key relationship: Interior angle + Exterior angle = 180° (at every vertex)

Angle Sums to Memorise

Polygon	n	Triangles	Angle Sum
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°
Nonagon	9	7	1260°
Decagon	10	8	1440°

Why? Split any polygon into $(n - 2)$ triangles from one vertex.

Convex vs. Concave & Symmetry

Convex: All interior angles $< 180^\circ$.

Concave: At least one reflex angle ($> 180^\circ$); has a "dent".

Quick test: A rubber band around a convex polygon touches every vertex. If it skips one, the polygon is concave.

Symmetry of regular n -gons:

- n lines of symmetry
- Rotational symmetry of order n
- Angle of rotation = $\frac{360^\circ}{n}$

Tessellation: Which Regular Polygons Tile a Plane?

For a regular polygon to tessellate on its own, its interior angle must divide exactly into 360° .

Shape	Angle	$360^\circ \div \text{angle}$	Tiles?	Shape	Angle	$360^\circ \div \text{angle}$	Tiles?
Eq. triangle	60°	6	Yes	Pentagon	108°	3.33...	No
Square	90°	4	Yes	Octagon	135°	2.67...	No
Hexagon	120°	3	Yes				

Only three: equilateral triangle, square, and regular hexagon.

Common Traps & Pitfalls

- Regular $\neq$ equal sides only:** A rhombus has 4 equal sides but is **not** regular (angles differ). Both conditions needed!
- Interior vs. exterior mix-up:** Read the question carefully. They are different values; don't give one when the other is asked.
- Forgetting to find n via exterior angles:** The fastest route to n is often: exterior angle $= 180^\circ - \text{interior angle}$, then $n = 360^\circ \div \text{exterior angle}$.
- Algebra errors with angle sums:** Always check your angles add up to $(n - 2) \times 180^\circ$ at the end.
- Confusing diagonals formula:** It's $\frac{n(n-3)}{2}$, **not** $\frac{n(n-1)}{2}$. Each vertex connects to $n - 3$ others (not itself, not its 2 neighbours).

Quick Exam Shortcuts

Finding number of sides from an interior angle: Exterior angle $= 180^\circ - \text{interior angle}$, then $n = 360 \div \text{exterior angle}$. Faster than solving $\frac{(n-2) \times 180}{n} = \theta$.

Missing angle in an irregular polygon: Angle sum $-$ sum of known angles $=$ missing angle.

Polygons meeting at a point: Angles around a point sum to 360° . Set up: $\theta_1 + \theta_2 + \dots = 360^\circ$.

Interior-exterior ratio: If interior $= k \times$ exterior, then $(k + 1) \times \text{ext} = 180^\circ$, so $\text{ext} = \frac{180}{k+1}$, then $n = \frac{360}{\text{ext}}$.