

Percentages

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What is a Percentage?

$$p\% = \frac{p}{100}$$

- **Fraction** → **%**: divide, then $\times 100$
- **%** → **Fraction**: write over 100, simplify
- **Decimal** → **%**: $\times 100$ (shift 2 right)
- **%** → **Decimal**: $\div 100$ (shift 2 left)

Mental shortcuts:

- 10%: $\div 10$ 5%: halve the 10%
- 1%: $\div 100$ 25%: $\div 4$

Common Conversions to Memorise

Fraction	Decimal	%	Fraction	Decimal	%
$\frac{1}{2}$	0.5	50%	$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%	$\frac{1}{10}$	0.1	10%
$\frac{3}{4}$	0.75	75%	$\frac{1}{8}$	0.125	12.5%
$\frac{1}{3}$	$0.3\bar{3}$	$33.\bar{3}\%$	$\frac{2}{3}$	$0.6\bar{6}$	$66.\bar{6}\%$

All Key Formulas

Skill	Formula	Key Rule
% of a quantity	$\frac{p}{100} \times Q$	Decimal $\times$ quantity
Express as %	$\frac{\text{Part}}{\text{Whole}} \times 100$	Same units. Part on top, whole on bottom
Increase / Decrease	Original $\times$ Multiplier	$+p\%$: $\times (1 + \frac{p}{100})$ $-p\%$: $\times (1 - \frac{p}{100})$
% Change	$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100$	Divide by ORIGINAL (not new)
Reverse %	$\frac{\text{Given Value}}{\text{Multiplier}}$	Find what % the given value represents, then $\div$
% Profit / Loss	$\frac{\text{Profit or Loss}}{\text{Cost Price}} \times 100$	Divide by COST PRICE (not selling price)

Multiplier Quick Reference

Situation	Mult.	Reverse by
Increase by 5%	$\times 1.05$	$\div 1.05$
Increase by 10%	$\times 1.10$	$\div 1.10$
Increase by 20%	$\times 1.20$	$\div 1.20$
Increase by 25%	$\times 1.25$	$\div 1.25$
Decrease by 10%	$\times 0.90$	$\div 0.90$
Decrease by 15%	$\times 0.85$	$\div 0.85$
Decrease by 20%	$\times 0.80$	$\div 0.80$
Decrease by 25%	$\times 0.75$	$\div 0.75$

Successive % Changes

Apply multipliers **one at a time** to the updated value.

Successive changes $\neq$ their sum!

$$Q \times m_1 \times m_2 \neq Q \times (m_1 + m_2)$$

Profit & Loss

$$\% \text{ Profit} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100 \quad \% \text{ Loss} = \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100$$

Divide by **Cost Price (CP)**, never the selling price.

★ SOS: Common Mistakes to Avoid ★

Mistake 1: Wrong denominator in % Change

- ✗ Dividing by the *new* value
- ✓ Always divide by the **original** value

Correct: $\frac{57.60 - 48}{48} \times 100 = 20\%$ NOT $\frac{57.60 - 48}{57.60}$

Mistake 2: Reverse % — subtracting the %

- ✗ $598 - 30\% = 598 - 179.40 = \$418.60$
- ✓ $\frac{598}{1.30} = \$460$

The % was applied to the *original*. Always divide by the multiplier.

Mistake 3: Wrong order — Part and Whole swapped

- ✗ $\frac{175}{56} \times 100 = 312.5\%$
- ✓ $\frac{56}{175} \times 100 = 32\%$

Part goes on top; whole goes on bottom.

Mistake 4: Adding successive %s together

- ✗ -25% then $-12\% = -37\%$ reduction
- ✓ $0.75 \times 0.88 = 0.66 =$ a 34% reduction

Multipliers must be *multiplied* together, not added.

Mistake 5: Profit/Loss — wrong denominator

- ✗ Dividing by the *selling* price
- ✓ Always divide by the **cost price**

$$\frac{34}{85} \times 100 = 40\% \text{ profit} \quad \text{NOT} \quad \frac{34}{119}$$

Mistake 6: Units not matched

- ✗ $\frac{750}{2.5} \times 100 = 30,000\%$

- ✓ Convert first: 2.5 l = 2500 ml, then $\frac{750}{2500} \times 100 = 30\%$

Keyword Spotting in Exam Questions

Exam phrase	What to do
“Find $x\%$ of ...”	Decimal $\times$ quantity
“What percentage is ... of ...”	Part $\div$ whole $\times 100$ (convert units first)
“Increase / decrease by $x\%$ ”	Multiply by multiplier ($1 \pm \frac{p}{100}$)
“Percentage change / increase / decrease”	$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100$; original on the bottom
“Find the original price / value”	Reverse %: given $\div$ multiplier
“Percentage profit / loss”	$\frac{\text{Profit or Loss}}{\text{Cost Price}} \times 100$
“Show that ...” (algebra)	Let original = P ; apply multipliers, expand, simplify

Checking Your Answers

- **% of a quantity:** check that answer $\div$ original = decimal
- **% Change:** re-apply the multiplier; the result should be the new value
- **Reverse %:** multiply your original by the multiplier — you should get back the given value
- **Profit/Loss:** a profit means $SP > CP$; a loss means $SP < CP$
- **Sanity check:** a 20% increase must give a number *larger* than the original; a decrease must give one *smaller*

Reverse % Step-by-Step

1. Decide whether the given value is **after an increase or decrease**
2. Calculate what % of the original the given value represents:
After $+p\%$: given = $(100 + p)\%$ of original
After $-p\%$: given = $(100 - p)\%$ of original
3. Convert that % to a decimal (the multiplier)
4. **Divide** the given value by the multiplier
5. Check: original $\times$ multiplier = given value? ✓