

Factorisation

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▷ What is Factorisation?

Writing an expression as a **product of its factors** — the reverse of expanding.

$$\begin{array}{ccc} 5(x+3) & \xrightarrow{\text{Expand}} & 5x+15 \\ \text{Factorised} & & \text{Expanded} \end{array}$$

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Always check: expand your answer to verify!

◇ Common Factor (HCF)

Find the **largest factor** in every term; write it outside the bracket. **Always do this first!**

- $10x + 35 = 5(2x + 7)$
- $4y^3 - 12y^2 = 4y^2(y - 3)$
- $8m^2n + 12mn^2 = 4mn(2m + 3n)$

Check both numbers **and** variables for HCF.

★ Factorising Quadratics (The Core Methods)

Form	Method
$x^2 + bx + c$	Find p, q : $p \times q = c$, $p + q = b$ Write $(x + p)(x + q)$
$ax^2 + bx + c$	Find p, q : $p \times q = ac$, $p + q = b$ Split middle term, group
4 terms	Group in pairs, factor each, take out the common bracket

Sign rules for $x^2 + bx + c$:

c	b	Signs of p, q
+	+	Both +
+	-	Both -
-	+	+ (larger), -
-	-	+, - (larger)

"Product is c , sum is b ."

□ Special Factorisation Patterns

Difference of Two Squares (DOTS):

$$a^2 - b^2 = (a + b)(a - b)$$

- $x^2 - 81 = (x + 9)(x - 9)$
- $25a^2 - 4b^2 = (5a + 2b)(5a - 2b)$

$a^2 + b^2$ **does NOT factorise!**

Perfect Square Trinomials:

$$a^2 \pm 2ab + b^2 = (a \pm b)^2$$

- $x^2 + 16x + 64 = (x + 8)^2$
- $9x^2 + 30x + 25 = (3x + 5)^2$

Check: first & last are squares, middle = $2ab$

△ Common Traps & Pitfalls

1. **Not fully factorised:**
 $18x^2 + 12x = 6(3x^2 + 2x) \times 6x(3x + 2) \checkmark$
2. **Sign errors with negatives:**
 $-4(b + 3) = -4b - 12 \checkmark$
3. **Dividing by x loses $x = 0$:**
Factorise: $x(x + 7) = 0 \checkmark$
4. $a^2 + b^2$ **cannot** be factorised!

✓ Quick Exam Strategy

- **Step 1:** Common factor out first
- **Step 2:** Count terms —
 $2 \rightarrow$ DOTS $3 \rightarrow$ Quadratic $4 \rightarrow$ Group
- **Step 3:** Can factors split further?
- **"Solve by factorising"** $\rightarrow$ set = 0,
then $AB = 0 \Rightarrow A = 0$ or $B = 0$
- **State reasons. Expand to check!**